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GRAVITATIONAL FIELD OF THE SCALARLY CHARGED MASS IN THE LOBACHEVSKI SPACE

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A variant of the Chernikov gravity theory with two connections and one metric, in which the background connection describes the Lobachevski space, is treated. A localized source of static, space spherically symmetric gravitational and massless scalar fields is present. An exact external solution of the problem is given. The result is yalid for the Rosen bimetric general relativity. The transition to the Einstein theory exists when the Lobachevski constant k tends to infinity.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Гравитационное поле скалярного заряда на фоне пространства Лобачевского

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В варианте теории Черникова с двумя связностями, в котором фоновая связность задается по Лобачевскому, найдено точное внешнее решение задачи о гравитационном поле локализованного источника статистических сферически-симметричных безмассовых скалярного и гравитационного полей. При стремлении константы Лобачевского $k \to \infty$ решение переходит в решение той же задачи в теории Эйнштейна. Результат справедлив для варианта биметрической общей теории относительности Розена.

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Some years ago Chernikov [1] has suggested a generalization of the Einstein theory, the theory with two connections but one metric. The main aim of this theory was to obtain a covariant generalization of the Einstein gravitational energy-momentum pseudotensor. For this aim it occurs necessary and sufficient to introduce the second (background) connection. A more early bimetric approach by Rosen [2] for the same aim proved, in this way, to be sufficient. Both theories have the general relativity as a limiting case, and this fact is to be taken into account if one appeals to an experiment.

The following variant [1] of the Chernikov theory is considered. First, when the gravitation is absent ($\kappa = 0$), the field connection $\Gamma^{\lambda}_{\mu\nu}$ is put equal to the background connection $\hat{\Gamma}^{\lambda}_{\mu\nu}$. Second, under the same condition, the static metric in the spherical space coordinates is

$$ds^2 = (cdt)^2 - dr^2 - \left(k \sinh \frac{r}{k}\right)^2 d\Omega^2, \quad d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2, \qquad (1)$$

so its spatial part describes the Lobachevski space (k is the Lobachevski constant) that tends to Euclidean one when $k \to \infty$. Third, the background connection chosen is Cristoffelian and defined by the metric (1). A consequence of the latter is the symmetry of the background connection Ricci tensor $\hat{R}^{\sigma}{}_{\mu\sigma\nu} \equiv \hat{R}_{\mu\nu} = \hat{R}_{\nu\mu}$ and field equations became

$$R_{\mu\nu} - \hat{R}_{\mu\nu} = \frac{8\pi\kappa}{c^4}\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right), \quad T \equiv g^{\rho\sigma} T_{\rho\sigma}, \quad \mu = 0, 1, 2, 3, \tag{2}$$

κ is the Newton gravity constant, $T_{\mu\nu}$ is the Einstein material tensor. Note that these equations coincide with equations of the Rosen bimetric general relativity with the background metric (1). In both theories, in analogy with general relativity (GR), the equation $\nabla_\mu T^{\mu\nu} = 0$ holds (∇_μ is the covariant derivative with respect to the field connection). Now, the consequence of it in the GR limit will be the De Donder (coordinate) harmonicity condition. For the static spherically symmetric space the equations (2) have been solved by Chernikov [1] when a concentrated massive source of the gravitational field is present. The interval has the form

$$ds^2 = V^2(r)(cdt)^2 - F^2(r)dr^2 - H^2(r)d\Omega^2, \tag{3}$$

and the exact external (i.e., $T_{\mu\nu} = 0$, $r \neq 0$) solution is of the form

$$V^2(r) = F^{-2}(r) = P^{-2} \frac{\sinh\frac{r - \hat{r}}{k}}{\sinh\frac{r + \hat{r}}{k}}, \quad P = \exp\left(-\frac{\hat{r}}{k}\right), \tag{4}$$

$$H(r) = Pk \sinh\frac{r + \hat{r}}{k}, \quad \frac{k}{2} \sinh\frac{2\hat{r}}{k} = \frac{\kappa m}{c^2}, \tag{5}$$

here m is the mass of the system. When $k \to \infty$ it turns into the Schwarzschild solution written in the spherical space coordinates that are conventionally related to the rectilinear harmonic coordinates x^i [3] , and $\hat{r} \to r_0 = \frac{\kappa m}{c^2}$. The relation is ordinary: $x^1 = r \sin\theta \cos\phi$, $x^2 = r \sin\theta \sin\phi$, $x^3 = r \cos\theta$.

Now we consider an analogous static problem when a localized source of the gravitational and massless scalar fields is present. For the scalar field potential U we assume the simplest generalization of the Schrödinger (Klein–Gordon–Fock) equation that outside the source gives

$$\nabla^\sigma \nabla_\sigma U = -F^{-2}\left[U'' + \left(\ln\frac{H^2 V}{F}\right)' U'\right] = 0, \tag{6}$$

the prime denotes d/dr. The static solution of it is

$$U' = -G\frac{F}{H^2 V} \tag{7}$$

that in the limit $\kappa \to 0$ should give $U' \to -G/r^2$, here G is the scalar constant. The corresponding scalar field material tensor is

$$T^{(sc)}_{\mu\nu} = -\frac{1}{4\pi}\left(\nabla_\mu U \nabla_\nu U - \frac{1}{2} g_{\mu\nu} \nabla_\sigma U \nabla^\sigma U\right). \tag{8}$$

By using (7) and (8), the field equations (2) give

$$\left(\frac{H^2 V'}{F}\right)' = 0, \tag{9}$$

$$\left(\frac{HH'V}{F}\right)' - FV\cosh\frac{2r}{k} = 0, \tag{10}$$

$$H'' - \frac{H}{k^2} - H'\frac{(FV)'}{FV} = -\frac{\kappa G^2}{c^4}\frac{F^2}{V^2 H^3}. \tag{11}$$

Before solving this system, let us recall that the searched solution in the limit $k \to \infty$ should turn into the solution of the same problem in GR in harmonic coordinates (more precisely, in spherical space coordinates, related to the rectilinear ones). I know this solution, it is as follows [4]:

$$V^2 = F^{-2} = \left(\frac{r-\tilde{\alpha}}{r+\tilde{\alpha}}\right)^\beta, \quad H^2 = (r+\tilde{\alpha})^{1+\beta}(r-\tilde{\alpha})^{1-\beta},$$

$$U = -\frac{G}{2\tilde{\alpha}} \ln \left|\frac{r-\tilde{\alpha}}{r+\tilde{\alpha}}\right|, \quad \text{i.e. } U' = -\frac{G}{r^2-\tilde{\alpha}^2}, \tag{12}$$

$$\tilde{\alpha} \equiv \frac{1}{c^2}\sqrt{\kappa^2 m^2 + \kappa G^2} = \frac{\kappa m}{\beta c^2}, \quad \beta \equiv \frac{\kappa m}{\sqrt{\kappa^2 m^2 + \kappa G^2}}.$$

Here we have explicitly introduced, into parameters, in distinction from the original, the mass m of the system and the scalar constant G defined by (7). The latter is not so indifferent how it could seem. This circumstance becomes clear, for instance, if we compare the solution (12) with the solution of the Nordström–Reissner problem for electric charge in harmonic coordinates [5] and when some limiting cases (κ, m, $G \to 0$) are considered.

Let us be convinced that (12) has been really expressed in harmonic coordinates as the authors do not indicate this fact. It is sufficient for this to use the consequence for space coordinates of the De Donder harmonicity condition ([6], Eq.(57.08)) (the rime coordinate is obviously harmonic)

$$\frac{1}{FV}\frac{d}{dr}\left(\frac{H^2 V}{F}\right) = 2r \tag{13}$$

that is obviously fulfilled. Now let us throw a glance at (4), (5) and at (12) and try to simply guess the solution in the form

$$V^2 = F^{-2} = P^{-2}\left(\frac{\sinh\frac{r-\bar{r}}{k}}{\sinh\frac{r+\bar{r}}{k}}\right)^{\beta}, \quad P \equiv \exp\left(-\frac{\bar{r}}{k}\right),$$

$$H^2 = P^2 k^2 \left(\sinh\frac{r+\bar{r}}{k}\right)^{1+\beta}\left(\sinh\frac{r-\bar{r}}{k}\right)^{1-\beta}, \tag{14}$$

$$U' = -\frac{G}{k^2}\left(\sinh\frac{r-\bar{r}}{k}\right)^{-1}\left(\sinh\frac{r+\bar{r}}{k}\right)^{-1},$$

where $\bar{r}$ is the integration constant. So, we are only left to define this parameter. From Eq.(11) we have

$$\frac{k^2}{4}\sinh^2\left(\frac{2\bar{r}}{k}\right) = \frac{\kappa^2 m^2 + \kappa G^2}{c^4}. \tag{15}$$

Our solution (14), (15), when $G \to 0$, tends to the Chernikov solution (4), (5), but differs from it in the essence of singularity when $r = \bar{r}$, in accordance with the property of GR solution (12) [4].

The question of uniqueness of (14), (15) remains still open both by virtue of the condition $FV = 1$ used and other possible reasons similar to those in general relativity [7].

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